

The background of the slide is a photograph of a sandy beach. In the foreground, there are numerous small, rhythmic ripples in the sand, likely created by wind or water. The sand is a light tan color. In the middle ground, there are some small, dark, rocky outcrops or clumps of vegetation. The horizon line is visible in the distance, where the sand meets a calm blue body of water. The sky is a clear, bright blue with a few wispy white clouds near the horizon.

Turbulence modulation in particle-laden flow

Bernard J. Geurts

**Multiscale Modeling and Simulation (Twente)
Anisotropic Turbulence (Eindhoven)**

Jastrzebia Gora, September 2008

Modulation of turbulence

Particles added to Navier-Stokes turbulence alter microscopic and macroscopic flow properties

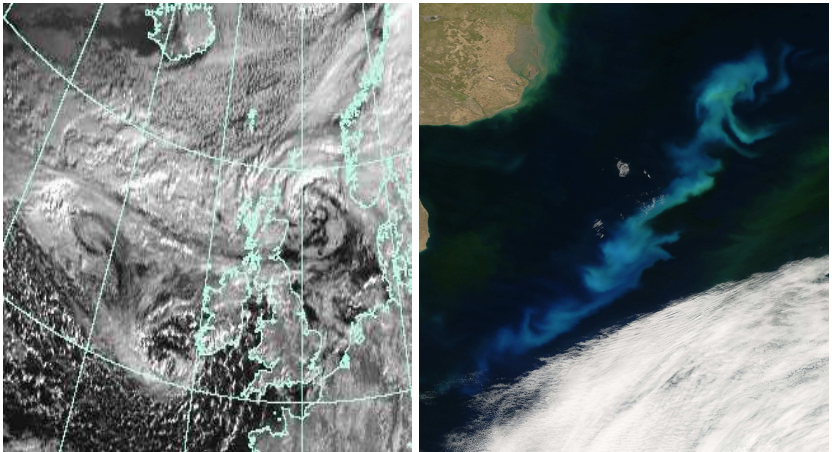
Also: e.g., due to rotation, stratification, combustion, ...

Muddy waters

Bend in Iruay river:

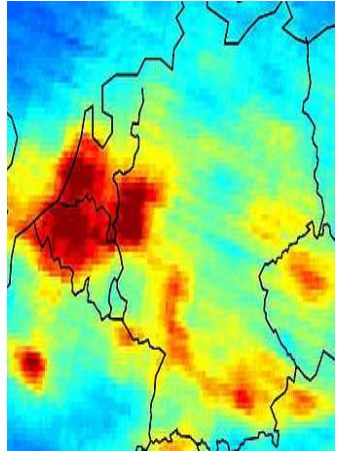


Environmental dispersion processes



European weather and Argentinian plankton bloom
Additional processes, e.g., phase transitions, active ‘swimming’

Dutch challenges



- Analysis of water motion, sediment transport and morphologic variability – long-term new coast?
- Spreading of pollutants - air-quality monitoring (NO_2)

Computational approach

Euler-Lagrange setting

- continuous liquid or gas phase
- discrete particulate phase (particles, droplets, aerosol, ...)

Integrate capability to simulate

- (a) turbulence (directly or after smoothing)
- (b) coupled, interacting point-particle system

Focus: development of methods and ‘canonical’ applications

Outline

- 1 **Turbulence and LES**
- 2 **Particle-laden turbulent flow**
- 3 **Concluding remarks**

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- 1 **Turbulence and LES**
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DNS and LES in one picture



Filtering Navier-Stokes equations

$$\partial_j u_j = 0 \quad ; \quad \partial_t u_i + \partial_j (u_i u_j) + \partial_i p - \frac{1}{Re} \partial_{jj} u_i = 0$$

Convolution-Filtering: filter-kernel G

$$\bar{u}_i = L(u_i) = \int G(\mathbf{x} - \xi) u(\xi) d\xi \quad ; \quad L(1) = 1$$

Large-eddy equations:

$$\partial_j \bar{u}_j = 0$$

$$\partial_t \bar{u}_i + \partial_j (\bar{u}_i \bar{u}_j) + \partial_i \bar{p} - \frac{1}{Re} \partial_{jj} \bar{u}_i = -\partial_j (\overline{u_i u_j} - \bar{u}_i \bar{u}_j)$$

Sub-filter stress tensor

$$\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$$

Some explicit subgrid models

Popular models:

- **Dissipation:** Eddy-viscosity models, e.g., Smagorinsky

$$\tau_{ij} \rightarrow -\nu_t S_{ij} = -(C_S \Delta)^2 |S| S_{ij} \quad ; \quad \text{effect} \quad \frac{1}{Re} \rightarrow \left(\frac{1}{Re} + \nu_t \right)$$

- **Similarity:** Inertial range, e.g., Bardina

$$\tau_{ij} \rightarrow [L, \Pi_{ij}](\bar{\mathbf{u}}) = \overline{\bar{u}_i \bar{u}_j} - \bar{u}_i \bar{u}_j$$

- **Mixed models ?**

$$m_{ij} = \textit{Bardina} + C_d \textit{Smagorinsky}$$

C_d via dynamic Germano-Lilly procedure

Convective modifications: Leray regularization

Alter convective fluxes:

$$\partial_t u_i + \overline{u}_j \partial_j u_i + \partial_i p - \frac{1}{Re} \Delta u_i = 0$$

LES template:

$$\partial_t \overline{u}_i + \partial_j (\overline{u}_j \overline{u}_i) + \partial_i \overline{p} - \frac{1}{Re} \Delta \overline{u}_i = -\partial_j (m_{ij}^L)$$

Implied Leray model

$$m_{ij}^L = L\left(\overline{u}_j L^{-1}(\overline{u}_i)\right) - \overline{u}_j \overline{u}_i = \overline{\overline{u}_j u_i} - \overline{u}_j \overline{u}_i$$

Requires approximate filter inversion

Regularization requires filter-inversion

Geometric series: repeated filtering

$$L^{-1} = (I - (I - L))^{-1} \rightarrow \sum_{n=0}^N (I - L)^n$$

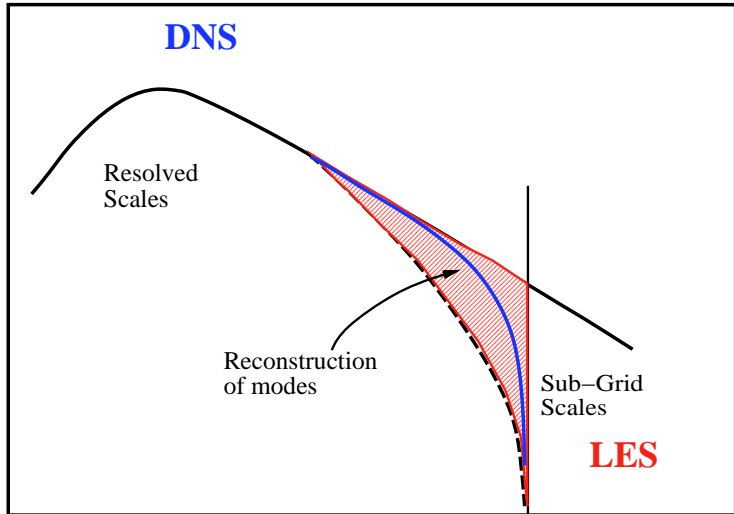
For example:

$$N = 0 : \quad u = L_0^{-1}(\bar{u}) = \bar{u}$$

$$N = 1 : \quad u = L_1^{-1}(\bar{u}) = \bar{u} + (I - L)\bar{u} = 2\bar{u} - \bar{\bar{u}}$$

$$\begin{aligned} N = 2 : \quad u &= L_2^{-1}(\bar{u}) = \bar{u} + (I - L)\bar{u} + (I - L)(I - L)\bar{u} \\ &= 3\bar{u} - 3\bar{\bar{u}} + \bar{\bar{\bar{u}}} \end{aligned}$$

Inversion and Energy distribution over Scales

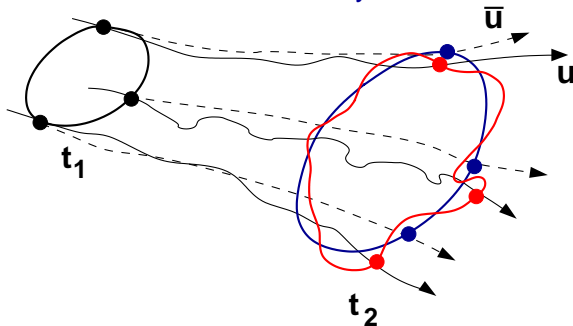


NS- α regularization

Kelvin's circulation theorem

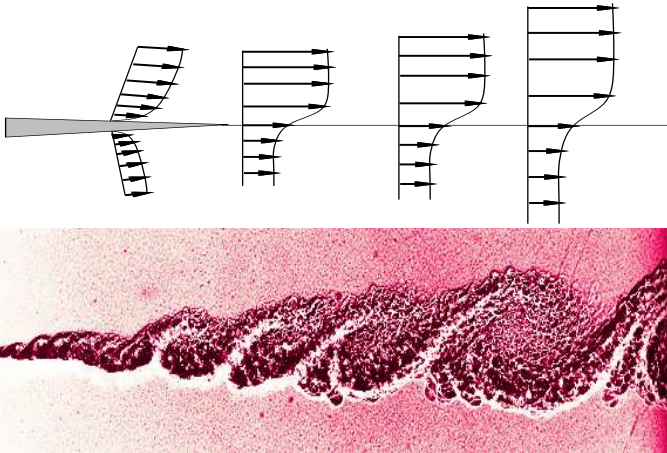
$$\frac{d}{dt} \left(\oint_{\Gamma(\mathbf{u})} u_j dx_j \right) - \frac{1}{Re} \oint_{\Gamma(\mathbf{u})} \partial_{kk} u_j dx_j = 0 \Rightarrow \text{NS - eqs}$$

Filtered Kelvin theorem extends Leray

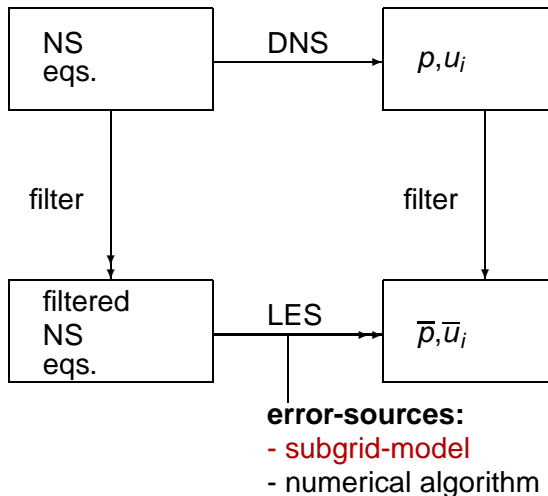


Obtain NS- α model by $\Gamma(\mathbf{u}) \rightarrow \Gamma(\bar{\mathbf{u}})$: Euler-Poincaré

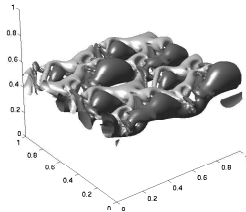
Mixing layer: testing ground for LES



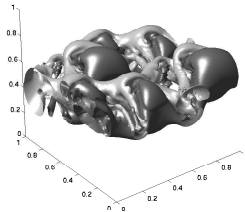
A posteriori LES testing



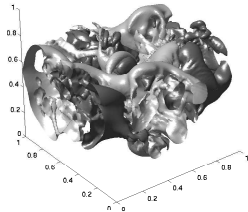
Mixing: spatial and temporal model



$t = 20$

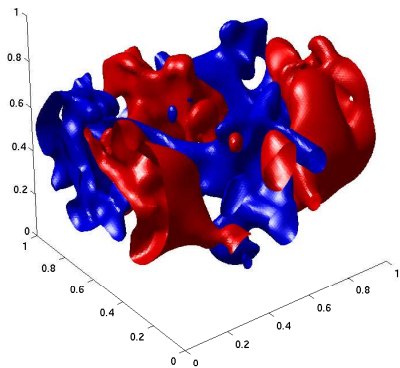
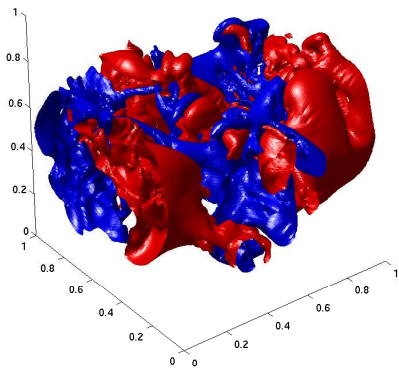


$t = 40$

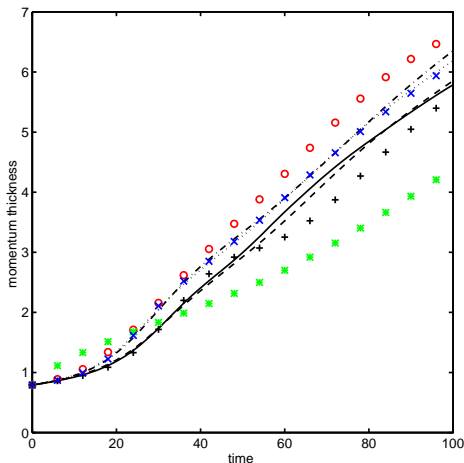


$t = 80$

Temporal at different $t \approx$ Spatial at different x

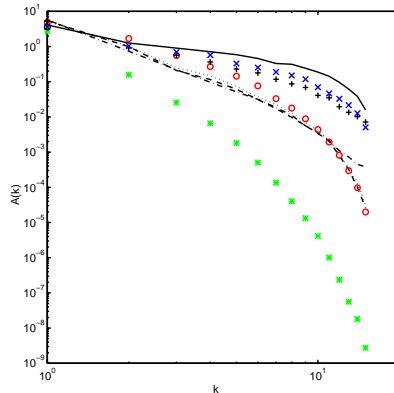


Mean flow - momentum thickness



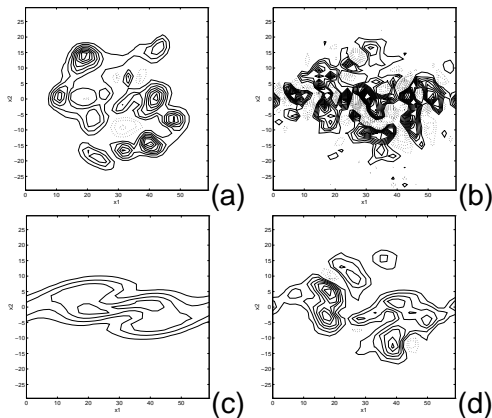
- Smagorinsky too dissipative
- Bardina, dynamic models preferred

Closer look: Streamwise energy spectrum



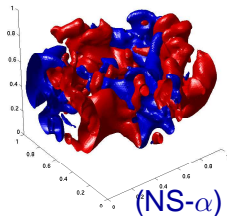
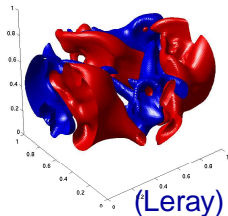
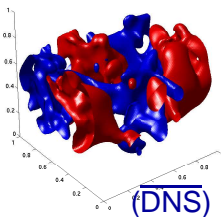
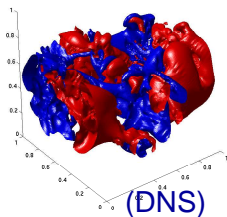
- Dissipation: Smagorinsky too much, Bardina not enough
- dynamic models quite acceptable
- **but:** 'middle range' wavenumbers much too low

Instantaneous snapshots of spanwise vorticity



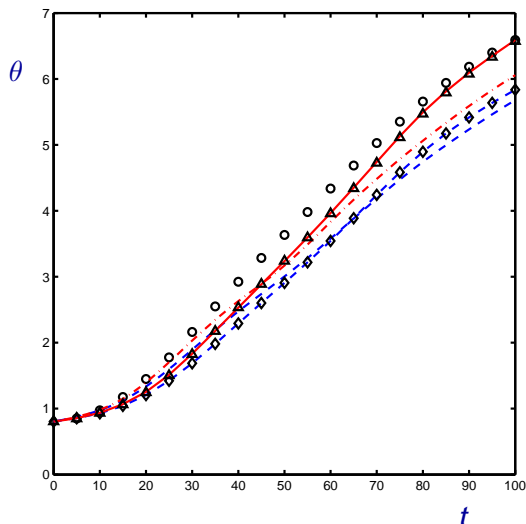
- a: $\overline{\text{DNS}}$, b: Bardina, c: Smagorinsky, d: dynamic
- Accuracy limited: regularization models better?

Leray and NS- α predictions: $Re = 50$, $\Delta = \ell/16$



Snapshot u_2 : red (blue) corresponds to up/down

Momentum thickness θ as $\Delta = \ell/16$



Filtered DNS (\circ)

Leray-model

32³: dash-dotted

64³: solid

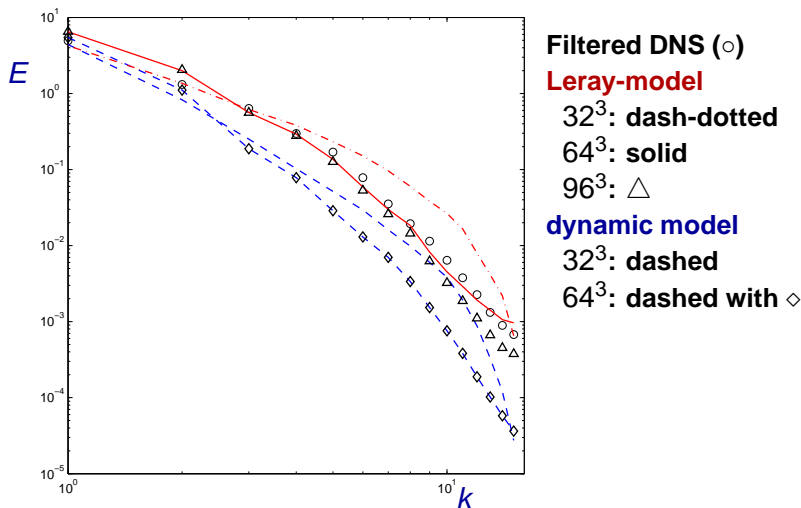
96³: \triangle

dynamic model

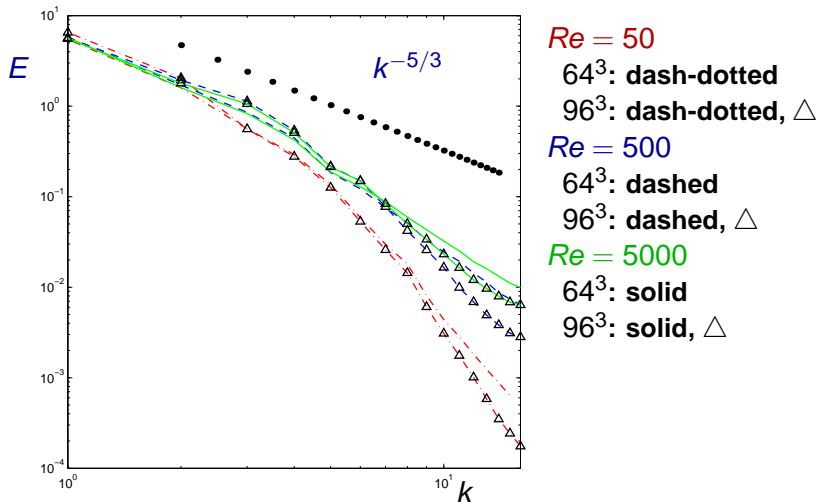
32³: dashed

64³: dashed with \diamond

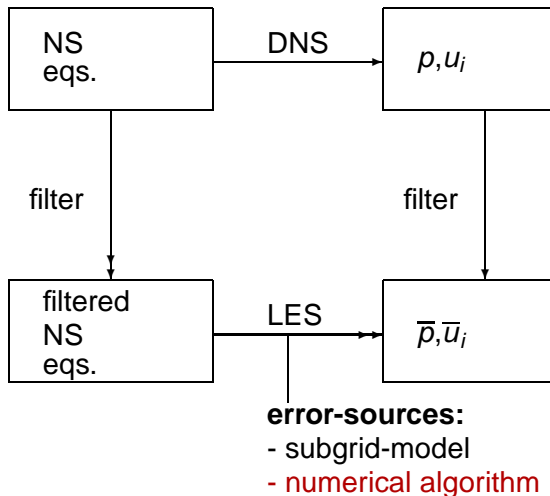
Streamwise kinetic energy E as $\Delta = \ell/16$



Robustness at arbitrary Reynolds number

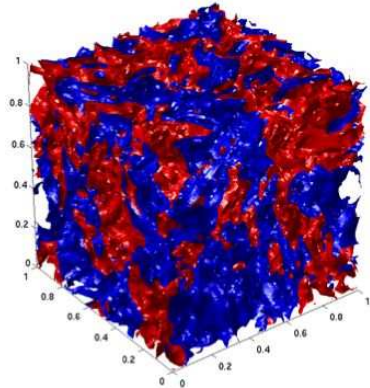
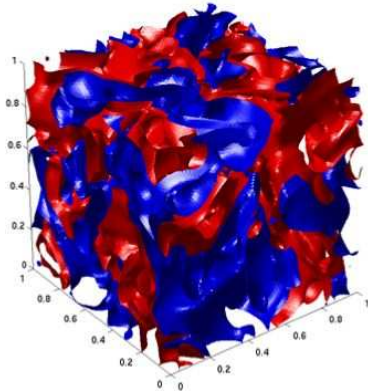


A posteriori LES testing

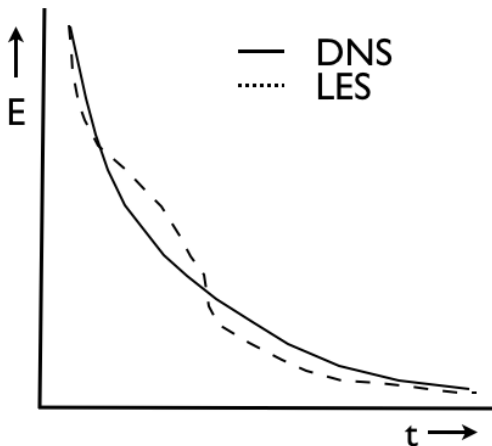


Smagorinsky fluid

Homogeneous decaying turbulence at $Re_\lambda = 50, 100$

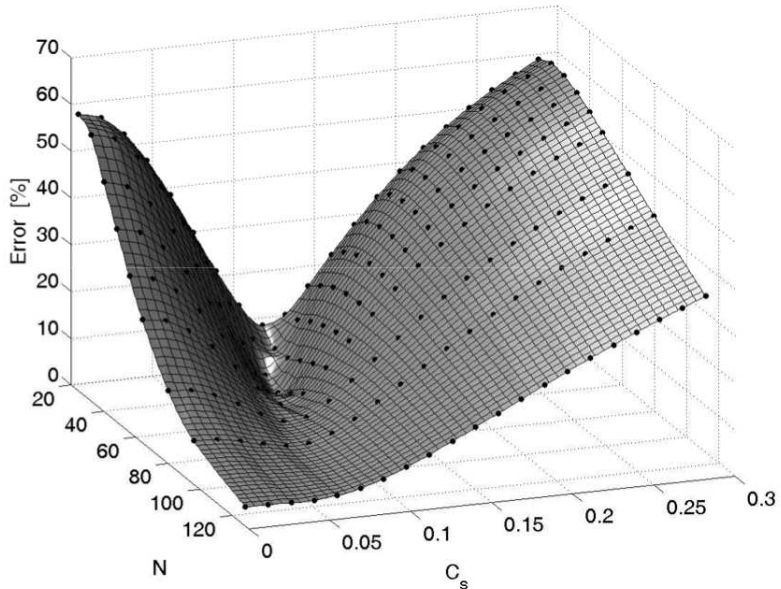


Decay of kinetic energy



Compare decay - quantify error

Total error-landscape: DNS-LES



Outline

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Process-engineering: Cracking of oil



Catalyst = dispersed particles

Safety?

Environment?

Chemical efficiency?

Are collisions important?

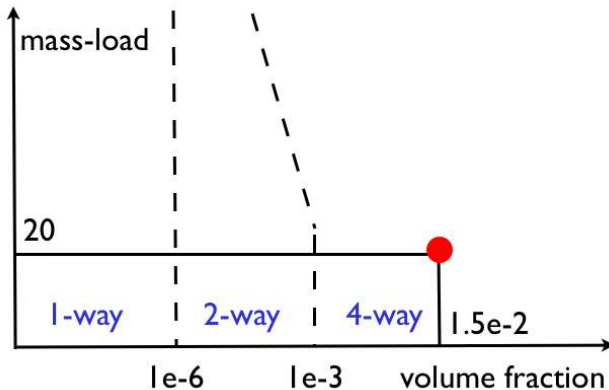
Do clusters form?

Operational Stability?

Computational approach: Euler-Lagrange

1-2-4 way coupling - Elghobashi

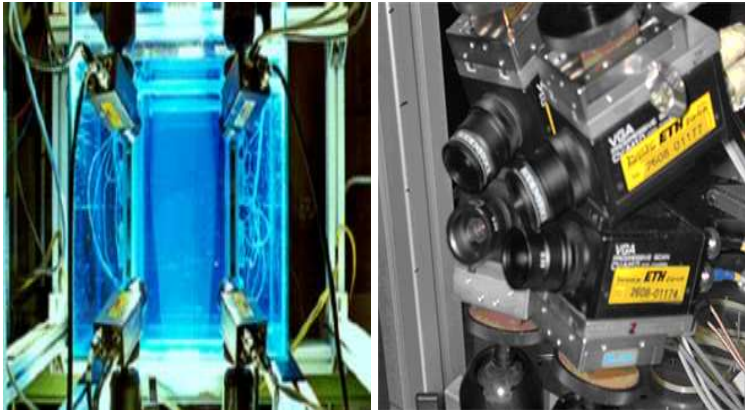
Volume fraction: $\psi = Nv_p/\Omega$: Mass-load: $m = \psi\rho_p/\rho_{fluid}$



Mass-load and volume fraction determine interaction regime

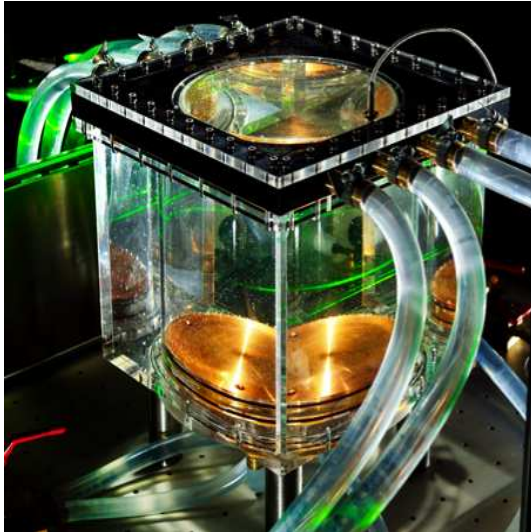
1-way

Test particles - PIV/PTV technique



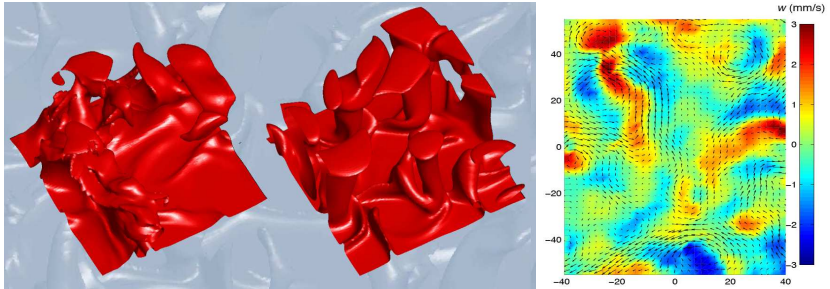
Small, light particles follow the flow precisely - no modulation
Recent developments: also measure temperature, ...
instrumented tracers ('smart particles')

Application: RRB



Collaboration TU/e: Rudie Kunnen, Herman Clercx

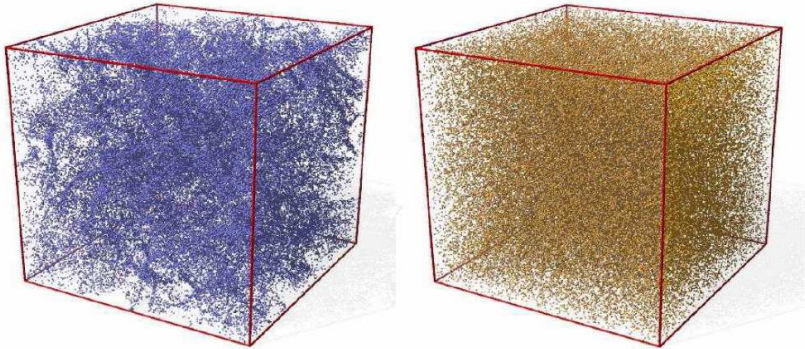
Rotating Rayleigh-Bénard problem



- **Temperature:** buoyancy dominated ($Ta = 0$ - left) and rotation dominated ($Ta > Ra$)
- **Stereo PIV measurement of (u, v, w) in horizontal plane**

Inertial effect - preferential clustering

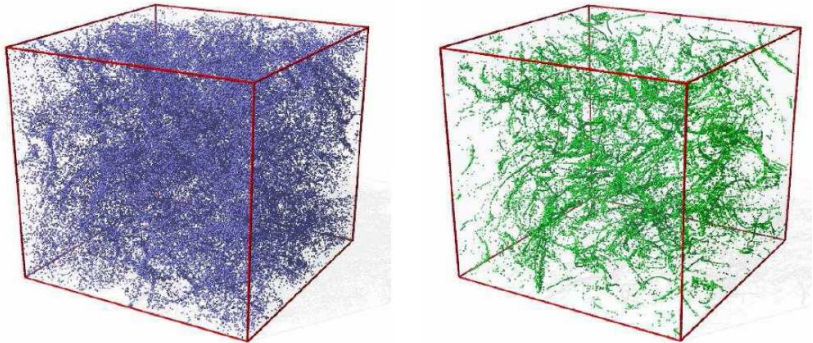
Motion of heavy particles in HIT



Heavy particles (left) display preferential clustering compared to test-particles (right)

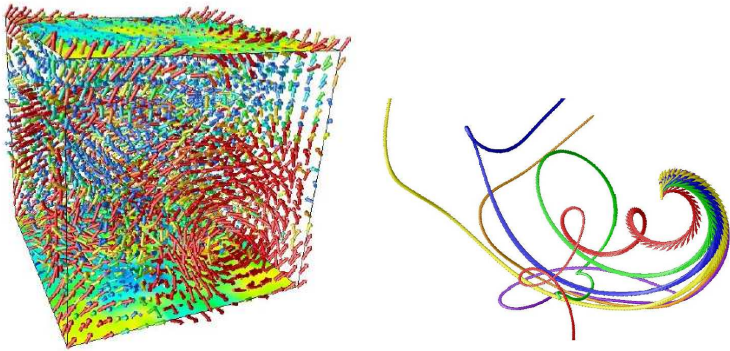
Inertial effect - preferential clustering

Motion of heavy and light particles in HIT



Heavy particles (left) are 'ejected' from high vorticity regions while bubbles (right) are staying close to pressure minima

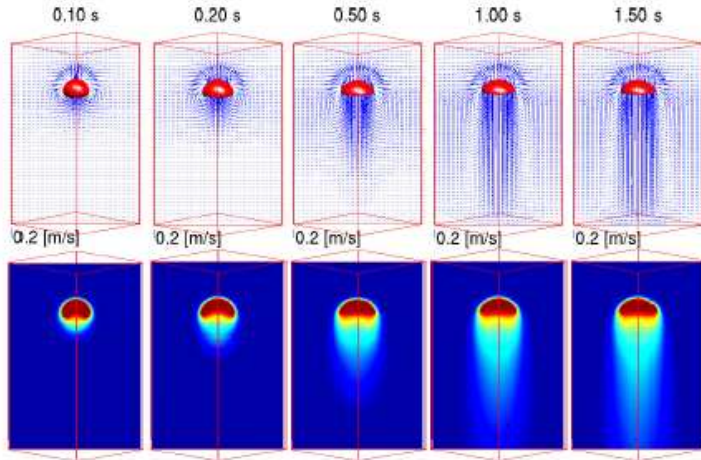
Heavy and light particles



Inertial particles respond differently to the same flow

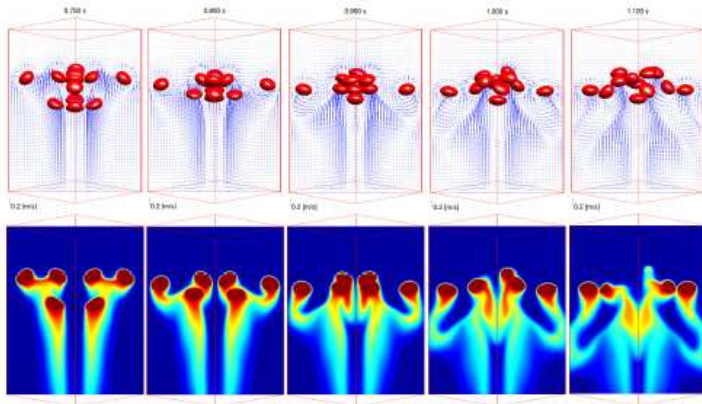
2-way

2-way coupling



Significant non-local flow modification

Spatially extended hydrodynamic interactions



VOF approach - interface tracking

In case many particles: interacting point-particle models

4-way

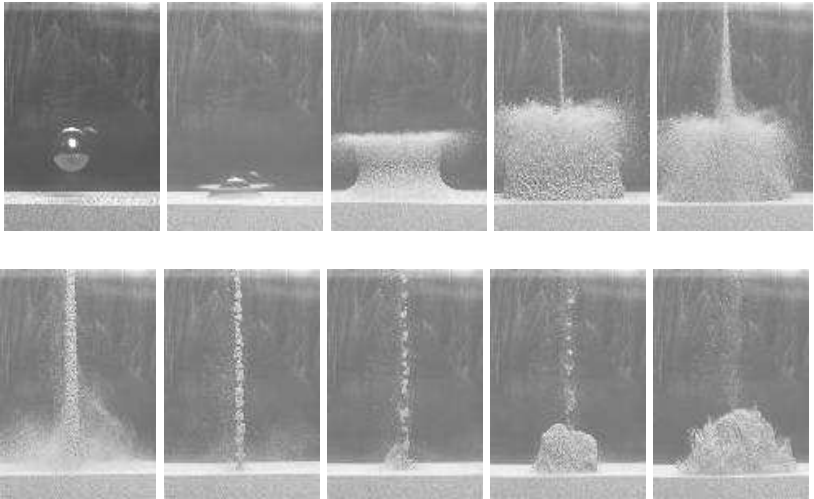
4-way coupling in 'dense' systems

'Large' volume fractions ($\gtrsim 10^{-3}$): (inelastic) collisions important

Dynamic self-organization:

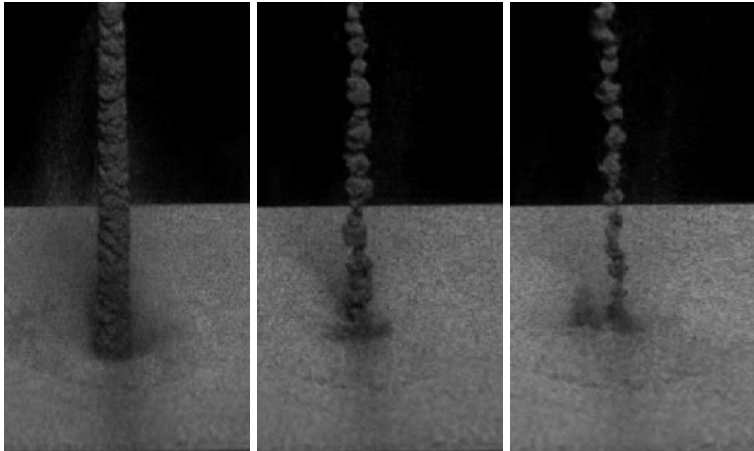
- microscopic dynamics expresses itself macroscopically

Sand-jet and granular clustering



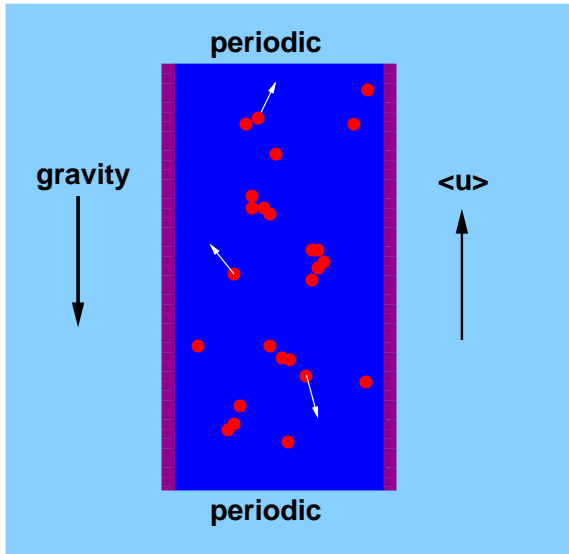
<http://www.tn.utwente.nl/pof/>

Close-up of granular clustering



Inelastic collisions yield progressive clustering

Gas-solid flow in vertical channel



Treatment of gas-phase

- **Fluid:** continuity, momentum

$$\begin{aligned}\partial_t(\rho\varepsilon) + \partial_j(\rho\varepsilon u_j) &= 0 \\ \partial_t(\rho\varepsilon u_i) + \partial_j(\rho\varepsilon u_i u_j) &= -\varepsilon\partial_i(p) + \partial_j(\varepsilon\sigma_{ij}) \\ &+ (\rho\varepsilon g + \varepsilon p_g)\delta_{i3} + f_i\end{aligned}$$

local porosity ε

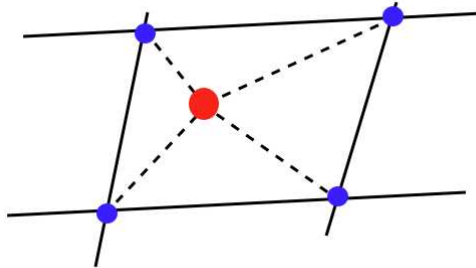
- Gravity ($\rho\varepsilon g$) and pressure gradient (εp_g) drive the flow
- Relative motion particles/fluid induces momentum transfer

Particles represent momentum sink

Momentum transfer (from drag force)

$$\mathbf{f}_{cell} = \frac{1}{V_{cell}} \int_{cell} \sum_{i=0}^{N_p} \frac{\mathbf{u} - \mathbf{v}_i}{\tau_p} D(\mathbf{r}_i - \mathbf{r}) dV$$

Function D distributes forces onto Eulerian grid



Treatment of solids phase

Discrete point-particles:

1. consider motion of single particle
2. add interactions (inelastic collisions)

Motion of small spherical particle

Maxey-Riley: (POF, 1982)

$$\begin{aligned} m_p \frac{dv_i}{dt} &= m_f \left(\frac{Du_i}{Dt} - \nu \nabla^2 u_i \right) \Big|_{\mathbf{x}(t)} \\ &- \frac{1}{2} m_f \frac{d}{dt} \left(v_i(t) - u_i(\mathbf{x}(t), t) \right) \\ &- 6\pi a \mu \left(v_i(t) - u_i(\mathbf{x}(t), t) \right) \\ &+ a \int_{-\infty}^t d\tau \frac{d/d\tau \{ v_i(\tau) - u_i(\mathbf{x}(\tau), \tau) \}}{[\pi \nu (t - \tau)]^{1/2}} \\ &+ (m_p - m_f) g_i \end{aligned}$$

Contributions: pressure gradient, added mass, Stokes drag, Basset viscous drag, buoyancy

Motion of small spherical particle

Maxey-Riley: (POF, 1982)

$$\begin{aligned} m_p \frac{dv_i}{dt} = & \\ & - 6\pi a \mu \left(v_i(t) - u_i(\mathbf{x}(t), t) \right) \\ & + (m_p - m_f) g_i \end{aligned}$$

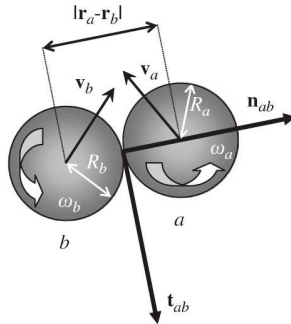
Consider: Stokes drag, buoyancy

Particle motion and collisions

Adding collisions: pp and pw

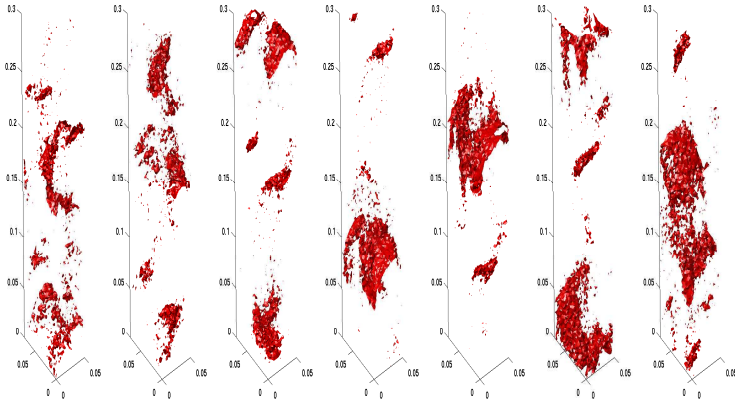
$$m \frac{d\mathbf{v}_i}{dt} = \alpha(Re_p) (\mathbf{u} - \mathbf{v}_i) + m\mathbf{g} + \mathbf{f}_i^{pp} + \mathbf{f}_i^{pw}$$

Inelastic collisions closed by 'restitution' coefficients for normal/tangential velocity, and friction (Hoomans et al., 1996)



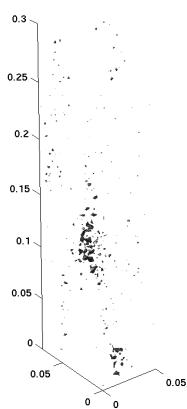
Number of particles: $N_p \approx 4.2 \cdot 10^5$ Volume-fraction $\approx 1.5\%$

Self-organization

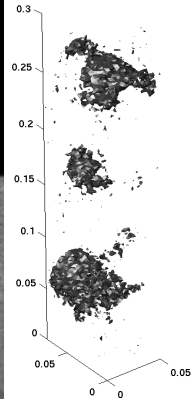
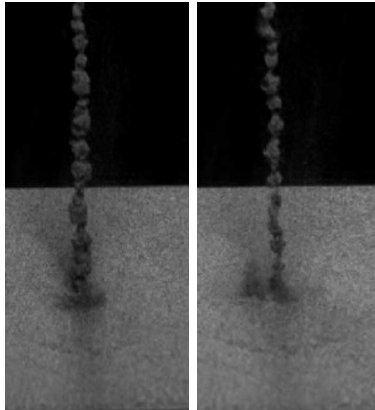


- Self-sustained generation/destruction of clusters [▶ Go](#)
- Mechanism: turbulent mixing and inelastic collisions

Clustering requires 4-way coupling



2-way



4-way

Particle-particle interactions \Rightarrow Clustering and Modulation

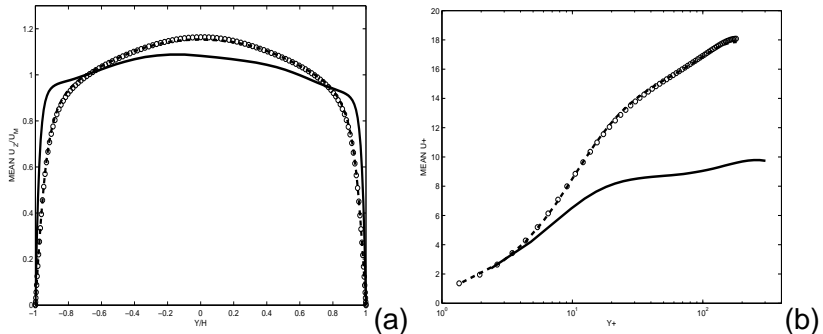
Simulation overview

Effects of collisions and particle-fluid interactions

Simulations considered:

- 1 clean channel
- 2 channel + particles, no collisions: 2-way coupling
- 3 channel + colliding particles: 4-way coupling

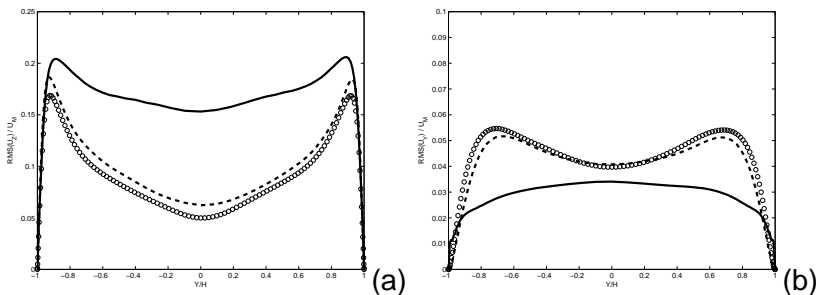
Mean fluid velocity: 4-way



Streamwise velocity $\langle u_z \rangle$: linear (a) and logarithmic (b)
Clean: DNS (\circ), LES (dash) – Particles: LES (solid)

- boundary layer thinner (Fig. a), skin-friction higher
- log-law changed: Von Kármán 'constant' higher (Fig. b)

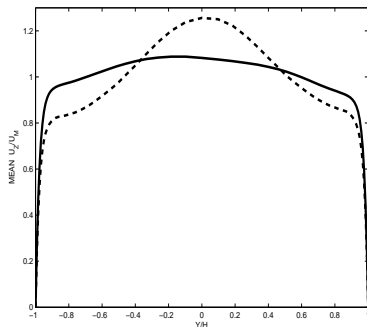
Turbulence intensities: clean and particles



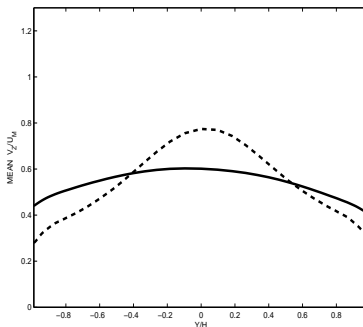
Streamwise (a) and normal (b) component
Clean: DNS (\circ), LES (dash) – Particles: LES (solid)

- Particles induce higher streamwise and lower normal/spanwise intensities

Mean flow velocities: 2-way and 4-way



(a)



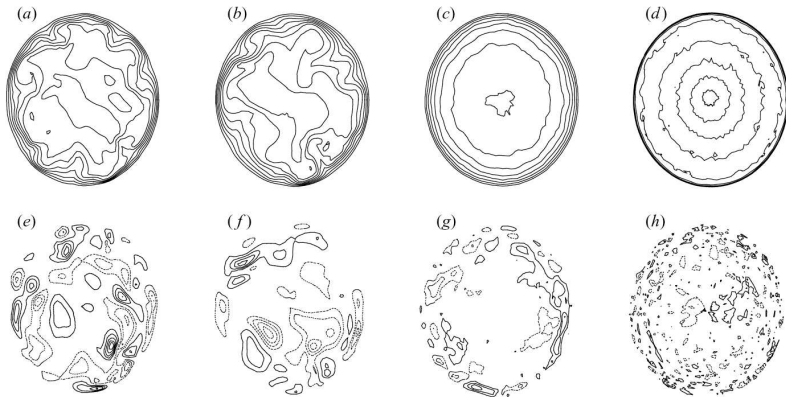
(b)

$\langle u \rangle$ of fluid (a) and $\langle v \rangle$ of solids phase (b)
4-way: solid and 2-way: dashed

- experimental observations: flattened profile
- 2-way gives rise to un-physical center-jet

Particle-laden flow in pipe geometry

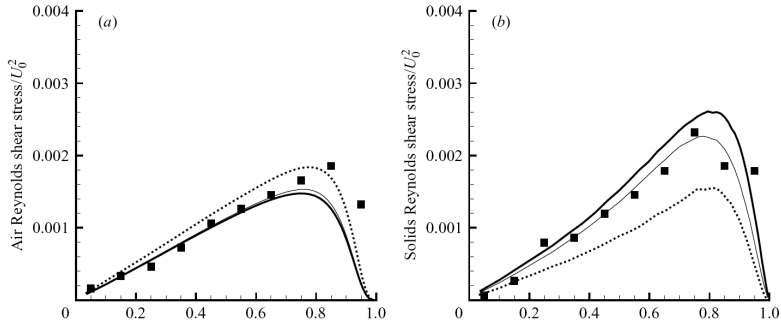
Apply methodology at different mass-loads:



Snapshots u_z (top) and u_θ (bottom) for (left to right) $m = 0$ (unladen), $m = 0.23$, $m = 1.1$, $m = 30$.

Vreman, JFM, 2007

Reynolds shear stress



Unladen (dotted) compared to particle-laden (with and without wall roughness model) at $m = 0.11$: experiment: Caraman et al. (2003).

Left: gas-phase, Right: solids phase at $m = 0.11$

Agreement less accurate in case of higher mass-load

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Concluding Remarks

LES:

- discussed concept - intuitive and mathematical modeling
- illustrated turbulent mixing
- hinted at error-sources and optimal parameter selection

Particle-laden flow:

- overview of regimes: 1-2-4
- considered Euler-Lagrange framework: continuum (DNS/LES) fluid with discrete (point) particles
- turbulent channel and pipe flow
- relevance of sub-filter motions for small particles: turbophoresis
- challenges in multiscale/multiphase systems

The background image is a photograph of a sandy beach. In the foreground, there are numerous small, rhythmic ripples in the sand, likely created by wind or water. The sand is a light tan color. In the middle ground, there are some larger, more irregular mounds of sand. In the background, the ocean is visible under a clear blue sky with a few wispy clouds. The overall scene is bright and sunny.

Turbulence modulation in particle-laden flow

Bernard J. Geurts

**Multiscale Modeling and Simulation (Twente)
Anisotropic Turbulence (Eindhoven)**

Jastrzebia Gora, September 2008